Abstract

The spectral function of pions interacting with a gas of nucleons and Δ_{33}-resonances is investigated using the formalism of Thermo Field Dynamics. After a discussion of the zero Δ-width approximation at finite temperature, we take into account a constant width of the resonance. Apart from a full numerical calculation, we give analytical approximations to the pionic spectral function including such a width. They are found to be different from previous approximations, and require an increase of the effective Δ-width in hot compressed nuclear matter. The results are summarized in an effective dispersion relation for interacting pions.

The Delta-Hole Model at Finite Temperature*

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1 Introduction

One of the main *experimental* goals of modern nuclear physics is the investigation of hot, compressed nuclear matter (HCNM). *Theoretical* descriptions of HCNM are mostly based on, or at least motivated by, methods of relativistic quantum field theory. However, within this framework it is a well established fact, that naive perturbation theory breaks down at finite temperature [1, 2]. This is due the modification of space-time symmetry in the presence of matter or a heat bath, i.e. to the absence of stable asymptotic states for the observable physical particles [3]. To overcome this problem, first of all one has to use one of several existing formulations of quantum field theory with 2×2 matrix-valued propagators.

While so far the Closed-Time-Path method (CTP) has been used to derive transport equations for heavy-ion collisions [4, 5], the model called Thermo Field Dynamics (TFD) offers technical advantages over CTP [6, 7]. Furthermore, within such a formulation, proper asymptotic conditions necessary for a perturbative approach can be defined only in TFD (see ref. [8] for details).

In this paper, TFD is applied to a system potentially interesting for the theoretical description of HCNM, namely to an interacting gas of nucleons, pions and Δ_{33}-resonances [9]. This Δ-hole model is often used [10, 11, 12], but in our view the analytical structure of the pion propagator is far from well understood.

We first describe the model as found in many applications, i.e. in the quasistatic zero-Δ-width approximation. Then a more realistic Lorentz-type spectral function for the Δ's is discussed, and the full pion propagator is calculated from a dispersion integral.

To obtain a systematic extension of the simple Δ-hole model to finite Δ-width and finite temperature, analytical results are given for the pion propagator by introducing an asymptotic expansion of the Δ spectral function to first order in its width. These results are simple enough to be used in further applications of the model, but still complete enough to contain the full information of the Δ-hole model.

It is discussed, how to fit this simplified model to the full calculation, and how good the approximations are at various temperatures, densities and momenta. Finally, the model is used to extract an "effective" pion dispersion relation in HCNM.

In the framework of TFD, the thermal instability of observable states can

be absorbed into a Bogoliubov transformation also for interacting systems. This Bogoliubov transformation defines stable quasi-particles which serve as basis for a perturbation expansion. It can be written in a single matrix form for the bosonic and the fermionic sector of the model as

$$\mathcal{B}_{B,F}(n) = \begin{pmatrix} (1 \pm n) & -n \\ \mp 1 & 1 \end{pmatrix} . \tag{1}$$

The Bogoliubov parameters n are the phase-space distribution functions for bosons or fermions. While one has some freedom in parameterizing this transformation, the above form was found to be the most useful in ref. [7], since it makes the propagators linear in n.

In the fermionic sector, i.e., for nucleons and Δ's, the n are Fermi-Dirac functions

$$n_{N,\Delta}(E) = \frac{1}{\mathrm{e}^{\beta(E-\mu_{N,\Delta})} + 1} , \tag{2}$$

with inverse temperature β. The baryon number in each small volume and hence the baryon density is a constant parameter of the calculations. Forgetting about the interaction at the moment, we express it in terms of bare "on-shell" energies

$$E_N(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2 + M_N^2} \quad \text{and} \quad E_\Delta(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2 + M_\Delta^2} , \tag{3}$$

and obtain as the baryon density without interaction

$$\begin{aligned} \rho_b^0 &= \rho_N^0 + \rho_\Delta^0 \\ &= 4 \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} n_N(E_N(\boldsymbol{p})) + 16 \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} n_\Delta(E_\Delta(\boldsymbol{p})) . \end{aligned} \tag{4}$$

In the fermion propagators we neglect anti-particle states. The resulting 2×2-matrices are

$$\begin{aligned} &S^{(ab)}_{N,\Delta}(p_0, \boldsymbol{p}) = -i \int dE\, \mathcal{A}_{N,\Delta}(E, \boldsymbol{p}) \times \\ &\tau_3 \left(\mathcal{B}_F(n_{N,\Delta}(E))\right)^{-1} \begin{pmatrix} \dfrac{1}{p_0 - E + i\epsilon} & \\ & \dfrac{1}{p_0 - E - i\epsilon} \end{pmatrix} \mathcal{B}_F(n_{N,\Delta}(E)) \end{aligned} \tag{5}$$

for both nucleons and Δ's. In this equation, $\mathcal{A}_{N,\Delta}$ is the spectral function of nucleons or Δ's, and $\mathcal{B}_F(n_{N,\Delta})$ are Bogoliubov matrices containing the thermal information as defined above, $\tau_3 = diag(1, -1)$.

2 Full pion propagator

The full pion propagator is the solution of a Schwinger-Dyson equation, with the free propagator and a self energy function (polarization function) as input. Each of these quantities is 2×2-matrix valued, and the full as well as the free propagator obey

$$D^{11} + D^{22} = D^{12} + D^{21} \,. \tag{6}$$

It is easy to see, that this implies for the polarization function

$$\Pi^{11} + \Pi^{12} + \Pi^{21} + \Pi^{22} = 0 \,. \tag{7}$$

In the following, we use results derived in ref. [8]. From the linear relation (7) between the matrix elements one obtains, that the matrix valued polarization function can be brought to triangular form by multiplication with bosonic Bogoliubov matrices $\mathcal{B}_B$

$$\begin{aligned} &\mathcal{B}_B(n_\pi(E))\,\tau_3\left(\Pi^{(ab)}\right) \quad (\mathcal{B}_B(n_\pi(E)))^{-1} \\ &= \quad \left(\begin{array}{ll} \Pi^{11}+\Pi^{12} & (1+n_\pi(E))\Pi^{12} - n_\pi(E)\Pi^{21} \\ 0 & \Pi^{11}+\Pi^{21} \end{array}\right) \end{aligned} \tag{8}$$

for *any* value of $n_\pi(E)$. The diagonal elements are the retarded and advanced components of the polarization

$$\begin{aligned} \Pi^R &= \Pi^{11} + \Pi^{12} \\ \Pi^A &= \Pi^{11} + \Pi^{21} \,. \end{aligned} \tag{9}$$

As one can derive, the off-diagonal components of Π are related by

$$\Pi^{12}(E, \boldsymbol{k}) - \mathrm{e}^{-\beta(E-\mu_\Delta+\mu_N)}\,\Pi^{21}(E, \boldsymbol{k}) = 0 \,. \tag{10}$$

Note, that this relation is quite similar (and indeed equivalent) to the Kubo-Martin-Schwinger boundary condition for propagators, which is equivalent to an equilibrium condition.

The full pion propagator therefore can be *diagonalized* by Bogoliubov matrices, if and only if $n_\pi(E)$ is chosen such that the off-diagonal element in (8) vanishes, i.e. by setting

$$n_\pi(E) = \frac{1}{\mathrm{e}^{\beta(E-\mu_\Delta+\mu_N)} - 1} \,. \tag{11}$$

Since $\mu_\pi = \mu_\Delta - \mu_N$ in our simple model, this is the pionic equilibrium distribution function, and thus the pion propagator is diagonal (separately for positive and negative energy states) only in case the pions have the same temperature as the baryons. For the results presented in this paper we have used $\mu_\pi = 0$, but the equations can be applied also to cases with nonzero pion chemical potential.

The full pion propagator is

$$
\begin{aligned}
D^{(ab)}(k_0, \boldsymbol{k}) \quad &= \int dE\, \mathcal{A}_\pi(E, \boldsymbol{k}) \;\times \\
&\left((\mathcal{B}_B(n_\pi(E)))^{-1} \begin{pmatrix} \dfrac{1}{k_0 - E + i\epsilon} & \\ & \dfrac{1}{k_0 - E - i\epsilon} \end{pmatrix} \mathcal{B}_B(n_\pi(E))\, \tau_3 \right. \\
- \quad & \left. \tau_3\, \mathcal{B}_B^T(n_\pi(E)) \begin{pmatrix} \dfrac{1}{k_0 + E - i\epsilon} & \\ & \dfrac{1}{k_0 + E + i\epsilon} \end{pmatrix} (\mathcal{B}_B^T(n_\pi(E)))^{-1} \right) .
\end{aligned} \tag{12}
$$

$\mathcal{A}_\pi(E, \boldsymbol{k})$ is a positive function, the spectral function of the pion field. Thus, apart from the task of determining this function, (12) already solves the problem of pion propagation in HCNM.

The spectral function is related to the imaginary part of the retarded and advanced propagator,

$$
D^{R,A}(E, \boldsymbol{k}) = \int dE'\, \mathcal{A}_\pi(E', \boldsymbol{k}) \left(\frac{1}{E - E' \pm i\epsilon} - \frac{1}{E + E' \pm i\epsilon} \right) , \tag{13}
$$

and the limit of free particles is recovered when

$$
\mathcal{A}_\pi(E, \boldsymbol{k}) \longrightarrow \delta(E^2 - E_\pi^2(\boldsymbol{k}))\Theta(E) , \tag{14}
$$

with the bare "on-shell" energy

$$
E_\pi(\boldsymbol{k}) = \sqrt{\boldsymbol{k}^2 + m_\pi^2} . \tag{15}
$$

For the interacting system, the propagator is best expressed through retarded and advanced components of the polarization since their analytical structure is well defined. One has

$$
\begin{aligned}
\Pi^{R,A}(E, \boldsymbol{k}) &= \mathrm{Re}\,(\Pi(E, \boldsymbol{k})) \mp i\, \pi \sigma(E, \boldsymbol{k}) \\
&= \int dE'\, \frac{\sigma(E', \boldsymbol{k})}{E - E' \pm i\epsilon} .
\end{aligned} \tag{16}
$$

The Schwinger-Dyson equation can be solved directly using this decomposition. In our formalism it is then diagonal, corresponding to two complex conjugate equations for the retarded and advanced propagator. The solution is

$$\mathcal{A}_\pi(E,\boldsymbol{k}) = \frac{\sigma(E,\boldsymbol{k})\,\Theta(E)}{\left(E^2 - E_\pi^2(\boldsymbol{k}) - \mathrm{Re}\,(\Pi(E,\boldsymbol{k}))\right)^2 + \pi^2\sigma^2(E,\boldsymbol{k})}\,. \tag{17}$$

In the limit of vanishing self energy the free-field result (14) is recovered.

We now use the fermion propagators (5) to obtain the polarization function for pions coupled to the fermions. Although we make a one-loop calculation, this does by no means imply a perturbative character of the calculation, since we can, in principle, use the *full* Green's function from the fermionic sector. All the interaction effects of the baryons are then plugged in their spectral functions, missing is only a possible vertex correction term. Such a vertex correction term can be parameterized as a medium dependence of the coupling constant.

In this generalized one-loop approximation, one has

$$\begin{aligned}\Pi_0^{(ab)}(E,\boldsymbol{k}) \;=\; -i\,s\,\boldsymbol{k}^2 \int\frac{d^4p}{(2\pi)^4}\,\Big[&g^{(a)}S_N^{(ab)}(p)g^{(b)}S_\Delta^{(ba)}(p-k)\\ &g^{(a)}S_N^{(ab)}(p)g^{(b)}S_\Delta^{(ba)}(p+k)\Big]\end{aligned} \tag{18}$$

where $s = 16/9$ is a spin degeneracy factor, $k = (E,\boldsymbol{k})$ in the integrand and

$$g^{(1)} = -g^{(2)} = \frac{f_{N\Delta}^\pi}{m_\pi}\,. \tag{19}$$

The numerical values for the coupling parameters and masses can be found in table 1.

From the definition (18) one can easily derive the relations (7) and (10). Inserting the fermion propagators given above then leads to a retarded pionic

$f_{N\Delta}^\pi$	g'	m_π	M_N	M_Δ	Γ
2	0.5	0.14 GeV	0.938 GeV	1.232 GeV	0.12 GeV

Table 1: Coupling constants and masses used in the calculations of this work.

polarization function

$$
\begin{aligned}
\Pi^R(E,\boldsymbol{k}) &= -2\pi\,\boldsymbol{k}^2\,\frac{16}{9}\left(\frac{f^\pi_{N\Delta}}{m_\pi}\right)^2 \int\frac{d^4p}{(2\pi)^4}\int dz_1\,dz_2\,\mathcal{A}_N(z_1,\boldsymbol{p}-\boldsymbol{k})\,\mathcal{A}_\Delta(z_2,\boldsymbol{p})\;\times \\
&\quad \left(\delta(p_0-E-z_1)\,\frac{n_N(z_1)}{p_0-z_2-i\epsilon}\;+\;\delta(p_0-z_2)\,\frac{n_\Delta(z_2)}{p_0-E-z_1+i\epsilon}\right) \qquad (20)\\
&+\;\text{the same expression with } E\to -E,\ \epsilon\to-\epsilon\ .
\end{aligned}
$$

The imaginary part of this expression determines the function $\sigma(E,\boldsymbol{k})$ as

$$
\begin{aligned}
\sigma(E,\boldsymbol{k}) &= \boldsymbol{k}^2\frac{16}{9}\left(\frac{f^\pi_{N\Delta}}{m_\pi}\right)^2\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\int dz\,\mathcal{A}_N(z,\boldsymbol{p})\;\times \\
&\quad \Big(\mathcal{A}_\Delta(z+E,\boldsymbol{p}+\boldsymbol{k})\,(\,n_N(z)-n_\Delta(z+E)\,) \qquad (21)\\
&- \quad \mathcal{A}_\Delta(z-E,\boldsymbol{p}+\boldsymbol{k})\,(\,n_N(z)-n_\Delta(z-E)\,)\Big)\ .
\end{aligned}
$$

This function rather than (20) is our starting point for the calculation of the full pion propagator, since it is numerically much easier to calculate. To include vertex corrections, one could then multiply $\sigma(E,\boldsymbol{k})$ by a form factor like in ref. [12], which however we will avoid here. The next step consists in calculating the dispersion integral (16).

Finally, we add another piece which has not been addressed so far. The perturbative expression for the pionic polarization tensor in the medium has to be corrected for the strong repulsive interaction at short distances. We use the phenomenological description

$$
\Pi_c(E,\boldsymbol{k}) = \frac{\boldsymbol{k}^2\,\Pi(E,\boldsymbol{k})}{\boldsymbol{k}^2-g'\,\Pi(E,\boldsymbol{k})}\ . \qquad (22)
$$

Note, that in HCNM g' also depends on the system parameters.

3 Quasistatic zero-width approximation

A first approximation to the Δ-hole model is obtained with the *free* spectral functions for nucleons and Δ's, ignoring the finite lifetime of the latter.

However, since this model is widely used, its critical discussion is necessary in this work. Hence we set for the nucleons

$$\mathcal{A}_N(E,\boldsymbol{p}) = \delta(E - E_N(\boldsymbol{p})) \; , \tag{23}$$

and neglect the width of the Δ-particle by using

$$\mathcal{A}_\Delta(E,\boldsymbol{p}) = \delta(E - E_\Delta(\boldsymbol{p})) \; . \tag{24}$$

The baryon density of the system is then given by the expression (4).

The quasistatic approximation follows from this by expanding the δ-function in the integrand in powers of $\boldsymbol{p}/M_N$. To lowest order, it corresponds to the neglection of recoil effects, i.e. by setting

$$\delta(E_\Delta(\boldsymbol{p}+\boldsymbol{k}) - E_N(\boldsymbol{p}) - E) \approx \delta(E_\Delta(\boldsymbol{k}) - M_N - E) \tag{25}$$

in the integrand. The δ-functions therefore do not affect the momentum integration, and one obtains

$$\begin{aligned}\sigma_{q0}(E,\boldsymbol{k}) \;&=\; \boldsymbol{k}^2 \frac{16}{9}\left(\frac{f^\pi_{N\Delta}}{m_\pi}\right)^2 \left\{\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\,\left(\,n_N(E_N(\boldsymbol{p})) - n_\Delta(E_\Delta(\boldsymbol{p}))\,\right)\right\} \times \\ &\qquad \Big(\delta(E - E_\Delta(\boldsymbol{k}) + M_N) \,-\delta(E + E_\Delta(\boldsymbol{k}) - M_N)\Big) \;. \end{aligned} \tag{26}$$

From eqn. (4) follows, that the factor in the curly brackets is equal to $\rho^0_N - 1/4\rho^0_\Delta$. This finding is in contrast to the equations used in [10, 11, 12], where the *baryon* density has been used instead. The difference is crucial for the model at higher temperature, where 50–60% of the baryon density are Δ's, hence the factor in the curly brackets is reduced by a factor ≈ 2 with respect to the baryon density.

The energy going into the Δ-production is

$$\omega_\Delta(\boldsymbol{k}) = E_\Delta(\boldsymbol{k}) - M_N = \sqrt{\boldsymbol{k}^2 + M^2_\Delta} - M_N \; . \tag{27}$$

Note, that in many applications the above expression is expand to lowest order in $\boldsymbol{k}$, thus worsening the asymptotic behavior as $\boldsymbol{k} \to \infty$ [10, 11, 12].

With this spectral decomposition of the polarization function, the dispersion integral (16) can be performed analytically, giving us the retarded polarization tensor in quasistatic zero-width approximation as

$$\Pi^R_{q0}(E,\boldsymbol{k}) = \boldsymbol{k}^2\frac{8}{9}\left(\frac{f^\pi_{N\Delta}}{m_\pi}\right)^2\left(\rho^0_N - \frac{1}{4}\rho^0_\Delta\right)\frac{\omega_\Delta(\boldsymbol{k})}{E^2 - \omega^2_\Delta(\boldsymbol{k}) + i\epsilon'} \; . \tag{28}$$

Here $\epsilon' = \text{sign}(E)\epsilon$ to obtain the proper (retarded) boundary conditions in time. The imaginary part is a δ-function, which gives zero contribution to the pion propagator.

The spectral function of the pions therefore consists of isolated poles $\omega_\pm$ obtained as the solution of

$$\omega^2 - E_\pi^2(\boldsymbol{k}) - \frac{\boldsymbol{k}^2\, C\, \omega_\Delta(\boldsymbol{k})}{\omega^2 - E_{N\Delta}^2(\boldsymbol{k})} = 0 \tag{29}$$

with

$$\begin{aligned} C &= \frac{8}{9}\left(\frac{f_{N\Delta}^\pi}{m_\pi}\right)^2 \left(\rho_N^0 - \frac{1}{4}\rho_\Delta^0\right) \\ E_{N\Delta}(\boldsymbol{k}) &= \sqrt{\omega_\Delta(\boldsymbol{k})\,(\omega_\Delta(\boldsymbol{k}) + g'\, C)}\,. \end{aligned} \tag{30}$$

Form factors depending only on the momentum $\boldsymbol{k}$ can be absorbed into C.

For simplicity the momentum arguments are suppressed from now on, and thus our solutions are

$$\omega_\pm^2 = \frac{1}{2}\left(E_{N\Delta}^2 + E_\pi^2 \pm \sqrt{\left(E_{N\Delta}^2 - E_\pi^2\right)^2 + 4\boldsymbol{k}^2 C\omega_\Delta}\right)\ . \tag{31}$$

The residues of the isolated poles in the propagator are

$$Z_\pm = \frac{1}{2}\left(1 \mp \frac{E_{N\Delta}^2 - E_\pi^2}{\sqrt{\left(E_{N\Delta}^2 - E_\pi^2\right)^2 + 4\boldsymbol{k}^2 C\omega_\Delta}}\right)\ , \tag{32}$$

and the retarded propagator itself reads

$$\begin{aligned} D_{q0}^R(E,\boldsymbol{k}) &= \frac{E^2 - E_{N\Delta}^2}{(E^2 - \omega_+^2 + i\epsilon')\,(E^2 - \omega_-^2 + i\epsilon')} \\ &= \frac{Z_+}{E^2 - \omega_+^2 + i\epsilon'} + \frac{Z_-}{E^2 - \omega_-^2 + i\epsilon'}\ . \end{aligned} \tag{33}$$

For the interpretation of these results we consider the free case, where for small momenta $E_\pi < E_{N\Delta}$, while at high momenta $E_\pi > E_{N\Delta}$ (see dotted lines in fig. 1).

This can be carried over to the interacting case, where for small momenta the spectral weight $Z_- > Z_+$, hence in this case ω_- is the energy of the pion. The other branch in this case is a Δ-hole excitation.

The roles are exchanged, when the momentum is larger. Then, ω_+ is the energy of the interacting pion with the larger strength Z_+, while ω_- is associated to the weaker Δ-hole branch.

The "crossover" point is determined by the momentum $k_\perp$ where the two residues are equal, i.e.

$$\begin{aligned} E_\pi^2(k_\perp) &= E_{N\Delta}^2(k_\perp) \\ \Leftrightarrow k_\perp^2 &= \left(\frac{M_\Delta^2 - M_N^2 - m_\pi^2}{2M_N - g'\,C} + M_N\right)^2 - M_\Delta^2\ . \end{aligned} \tag{34}$$

This value is marked in fig. 1 by a vertical line. Hence we can safely set

$$\begin{aligned} \text{pion energy}\ \ \omega_\pi &= \begin{cases} \omega_- & \text{if} \quad |\boldsymbol{k}| < k_\perp \\ \omega_+ & \text{if} \quad |\boldsymbol{k}| > k_\perp \end{cases} \\ \Delta\text{-hole energy}\ \ \omega_{N\Delta} &= \begin{cases} \omega_+ & \text{if} \quad |\boldsymbol{k}| < k_\perp \\ \omega_- & \text{if} \quad |\boldsymbol{k}| > k_\perp \end{cases} \quad . \end{aligned} \tag{35}$$

The energies $\omega_\pm$ are plotted in figure 1 (full lines), for the example value of 1.69 nuclear density as function of $|\boldsymbol{k}|$. The values of the coupling constants and masses used in the calculation are given in table 1.

Clearly the notion of a dispersion relation with a discontinuity, or *jump*, as implied by the above description, is somewhat unusual and requires further comment. We postpone this until we have discussed a way to include the width of the branches into the considerations.

In contrast to our derivation, it has been argued in ref. [10], that one may introduce the energies of the quasi-pion and the Δ-hole excitation as the two combinations

$$\begin{aligned} \Omega_\pi &= Z_-\omega_- + Z_+\omega_+ \\ \Omega_{N\Delta} &= (1 - Z_-)\omega_- + (1 - Z_+)\omega_+ \ = \ Z_+\omega_- + Z_-\omega_+\ . \end{aligned} \tag{36}$$

These energies are also plotted in figure 1 (dash-dotted lines) – but their choice is unjustified. They do not correspond to any physical excitation of the system under consideration.

To prove this we observe, that from the viewpoint of quantum field theory the renormalization of the wave function is done with the factors $\sqrt{Z_\pm}$, i.e., they are the coefficients of the fields in the dynamical map [13]. To leading

order and for fixed momentum, the dynamical map for the two interacting fields can be written explicitly as

$$\begin{aligned} \psi_+ &= \sqrt{Z_+}\,\mathcal{B}_B[E_\pi]\,\xi_\pi + \sqrt{Z_-}\,\mathcal{B}_B[E_{N\Delta}]\,\xi_{N\Delta} + \dots \\ \psi_- &= \sqrt{Z_+}\,\mathcal{B}_B[E_{N\Delta}]\,\xi_{N\Delta} - \sqrt{Z_-}\,\mathcal{B}_B[E_\pi]\,\xi_\pi + \dots\ . \end{aligned} \tag{37}$$

Here, ξ_π is the free pion Heisenberg field (thermal doublet, cf. [7]), transformed with the Bogoliubov matrix taken at the proper on-shell energy. $\xi_{N\Delta}$ is the operator creating a Δ-hole pair.

Hence, a $Z_\pm$-weighted sum of the eigenmode energies $\omega_\pm$ does not give the energy of a physical state, but up to factors $\sqrt{Z}$ inverts the above transformation. This can be seen immediately by calculating, with the same justification as can be given for eqn. (36), the averages of the *squared* energies. The results then are

$$\begin{aligned} Z_-\omega_-^2 + Z_+\omega_+^2 &= E_\pi^2 \\ Z_+\omega_-^2 + Z_-\omega_+^2 &= E_{N\Delta}^2\ , \end{aligned} \tag{38}$$

i.e., the *free* energies (dotted lines in figure 1). For all momenta the first choice (36) differs only very slightly from the free energies.

In summary of this comment one can state, that the $Z_\pm$ weighted energy average removes the interaction from the system. We therefore feel, that such an averaging process should be avoided: the resulting very small modification of the pion dispersion relation with respect to the free one can be obtained much easier, than by arguments about polarization functions.

4 Lorentz spectral function

The use of a free spectral function for the nucleons is justified by the great success of the quasiparticle concept in nuclear physics. It can easily be modified by introducing a density (and temperature) dependent effective nucleon mass, or collective effects due to nucleon-hole excitations [14]. For simplicity however we have kept the bare nucleon, and therefore use the spectral function as specified in (23).

The Δ_{33} *resonance* in contrast has a vacuum width of 120 MeV. Since this is comparable to the pion mass, it should not be neglected in any serious

computation. The width of the Δ spectral function peak also grows with momentum [15], but comparison with scattering data restricts it to about twice the above value in the momentum range of interest here [16].

Even less is known about its medium dependence [11], hence without a self-consistent calculation we feel safest to estimate the influence of a constant Δ-width on the pion spectrum. Such an approximation then clearly does not have the proper threshold behavior, but can easily be generalized to such case. For the following, we therefore use the spectral function

$$\mathcal{A}_\Delta(E,\boldsymbol{p}) = \frac{1}{2\pi}\,\frac{\Gamma}{(E-E_\Delta(\boldsymbol{p}))^2+(\Gamma/2)^2}\ . \tag{39}$$

With the above spectral functions, the actual baryon density instead of (4) becomes

$$\begin{aligned}\rho_b &= \rho_N^0+\rho_\Delta \\ &= 4\int\!\frac{d^3\boldsymbol{p}}{(2\pi)^3}\,n_N(E_N(\boldsymbol{p}))+16\int\!\frac{d^3\boldsymbol{p}}{(2\pi)^3}\int\! dE\,\mathcal{A}_\Delta(E,\boldsymbol{p})\,n_\Delta(E)\ .\end{aligned} \tag{40}$$

The imaginary part of the polarization function is

$$\begin{aligned}\sigma(E,\boldsymbol{k}) &= \boldsymbol{k}^2\frac{16}{9}\left(\frac{f^\pi_{N\Delta}}{m_\pi}\right)^2\int\!\frac{d^3\boldsymbol{p}}{(2\pi)^3}\,\mathcal{A}_\Delta(E_N(\boldsymbol{p})+E,\boldsymbol{p}+\boldsymbol{k})\ \times \\ &\qquad\qquad \Big(n_N(E_N(\boldsymbol{p}))-n_\Delta(E_N(\boldsymbol{p})+E)\Big) \\ &\quad - \ \text{the same expression with } E\to -E\ ,\end{aligned} \tag{41}$$

The complete momentum integral in the second part of (40) can be done analytically. For the imaginary part of the polarization tensor in eqn. (41) however, this only holds for the angular integration. Using $p=|\boldsymbol{p}|$ and $k=|\boldsymbol{k}|$ in the following, we obtain

$$\begin{aligned}\sigma(E,\boldsymbol{k}) &= \tfrac{2\boldsymbol{k}^2}{9\pi^3}\left(\tfrac{f^\pi_{N\Delta}}{m_\pi}\right)^2\int\! p^2\,dp\ [(\,n_N(E_N(p))-n_\Delta(E_N(p)+E)\,)\,I(E,k,p) \\ &\qquad -\ (\,n_N(E_N(p))-n_\Delta(E_N(p)-E)\,)\,I(-E,k,p)]\end{aligned} \tag{42}$$

with the function

$$I(E,k,p) = \int\! d\Omega_p\,\mathcal{A}_\Delta(E+E_N(p),\boldsymbol{p}+\boldsymbol{k})$$

$$
= \frac{1}{pk}\,\Theta(E_N(p)+E)\,\left[\frac{\Gamma}{2}\,\log\left(x^2+\left(\frac{\Gamma}{2}\right)^2\right)\right. \qquad (43)
$$
$$
\left.\left. -2(E_N(p)+E)\,\arctan\left(\frac{\Gamma}{2x}\right)\right]\right|_{x\,=\,w_-}^{x\,=\,w_+} .
$$

The boundaries to be inserted for x are

$$
w_\pm = E_\Delta(p \pm k) - E_N(p) - E\ . \qquad (44)
$$

Remaining for the calculation of σ is therefore the p-integration, which we perform numerically. It is followed by the calculation of the dispersion integral and by the subsequent correction according to eqn. (22). The resulting spectral function is plotted as function of temperature and energy for five different momenta in fig. 2 – 6.

Clearly, the spectral function has only one peak (as function of energy) at low momenta (fig.2 and 3) - corresponding to an almost free pion, with a momentum dependent width. At momenta in the vicinity of $k_\perp$ according to (34), this mode is strongly mixed with the Δ-hole excitation, and two peaks appear in the spectral function (cf. fig. 4).

At higher momenta, this mixing leads to the crossover of the two branches, and the diminishing of the peak at lower energies (fig. 5 and 6). It is worthwhile to note, that the Δ-hole peak vanishes with increasing temperature at all momenta. Hence at temperatures above 0.1 GeV, one cannot speak of the Δ-hole like excitation as a degree of freedom of the system: it is dissolved in the medium.

Our result is, that the location of the peaks corresponds quite well to the eigenmodes $\omega_\pm$ of the quasistatic zero-width approximation, cf. the dots in fig. 1. Hence the latter is useful even at finite temperature.

The same calculations have been performed with a Δ-width of 2Γ, which therefore allows to study the modifications introduced by a realistic parameterization of the Δ spectral function in terms of scattering data [16]. The results then are quantitatively, but not qualitatively different from those presented in figs. 2 – 6. They only give an even more pronounced smearing of the two spectral peaks at momenta around $k_\perp$ according to (34) (cf. fig. 4).

5 Expansion in lowest order Γ

The calculation of dispersion integrals with sufficient precision appears to be impractical for many purpose. Hence in the following we study a modification of the quasistatic zero-width approximation, which takes into account the finite width effects in a controlled fashion.

To this end, we start from eqn. (41) and perform a quasistatic approximation as before by neglecting the $\boldsymbol{p}$-dependence only in the spectral function of the Δ's. Care has to be taken, however, because of the rather awkward factor from the second distribution function. Hence we assume, that $\mathcal{A}_\Delta$ is still quite peaked around the bare energy value. Then we have approximately

$$\sigma_q(E,\boldsymbol{k}) = \boldsymbol{k}^2 \frac{16}{9} \left(\frac{f^\pi_{N\Delta}}{m_\pi}\right)^2 \left(\rho^0_N - \frac{1}{4}\rho^0_\Delta\right) \left(\mathcal{A}_\Delta(M_N+E,\boldsymbol{k}) - \mathcal{A}_\Delta(M_N-E,\boldsymbol{k})\right) . \tag{45}$$

Note, that still ρ^0_Δ appears in the coupling factor, rather than ρ_Δ as defined in (40).

Now we follow the approach from [17] and perform an asymptotic expansion of the Δ spectral function

$$\mathcal{A}_\Delta(E,\boldsymbol{p}) = \delta(E - E_\Delta(\boldsymbol{p})) - \frac{1}{2\pi}\frac{\partial}{\partial E}\frac{\mathcal{P}\,\Gamma}{E - E_\Delta(\boldsymbol{p})} . \tag{46}$$

Here, $\mathcal{P}$ denotes the principal value in case one integrates over the energy pole. Note, that this equation also holds in case Γ acquires an E, $\boldsymbol{k}$, T-dependence. Using eqn. (45), this yields

$$\begin{aligned}\sigma_{q1}(E,\boldsymbol{k}) \; &= \boldsymbol{k}^2 \frac{16}{9} \left(\frac{f^\pi_{N\Delta}}{m_\pi}\right)^2 \left(\rho^0_N - \frac{1}{4}\rho^0_\Delta\right) \Bigg(\delta(E - E_\Delta(\boldsymbol{k}) + M_N) \\ &- \delta(E - E_\Delta(\boldsymbol{k}) - M_N) + \frac{\Gamma\, E}{\pi}\frac{\omega_\Delta(\boldsymbol{k})}{\left(E^2 - \omega^2_\Delta(\boldsymbol{k})\right)^2}\Bigg)\end{aligned} \tag{47}$$

Only the δ-functions contribute to the dispersion integral (16), hence to first order in Γ the polarization function is

$$\Pi^R_{q1}(E,\boldsymbol{k}) \;=\; \boldsymbol{k}^2 C\,\frac{\omega_\Delta}{E^2 - \omega^2_\Delta} - \boldsymbol{k}^2 C\, i\,\frac{\omega_\Delta E\,\Gamma}{\left(E^2 - \omega^2_\Delta\right)^2} \tag{48}$$

$$\approx\; \boldsymbol{k}^2 C\,\frac{\omega_\Delta}{(E + i\Gamma/2)^2 - \omega^2_\Delta} \tag{49}$$

(before the g'-correction according to (22)). The pion propagator obtained with this approximation has two poles in the proper complex energy half plane, and is an analytical function in the other half plane.

Another approximation used frequently is obtained by simply shifting ω_Δ by $i\Gamma/2$ as in refs. [19, 12]:

$$\boldsymbol{k}^2 C \,\frac{\omega_\Delta - i\Gamma/2}{E^2 - (\omega_\Delta - i\Gamma/2)^2}\ . \tag{50}$$

However, in contrast to our derivation above, the pion propagator obtained with (50) has not the proper structure in the complex energy plane (see the comment below and fig. 7).

After the correction (22) is performed, the correct pionic spectral function obtained with the expression (49) is, to first order in Γ,

$$\mathcal{A}_\pi(E,\boldsymbol{k}) = \frac{\boldsymbol{k}^2 C}{\pi}\,\frac{\Gamma\, E\,\omega_\Delta\,\Theta(E)}{(E^2-\omega_+'^2)^2\,(E^2-\omega_-'^2)^2 + \Gamma^2\,E^2\,(E^2-E_\pi^2)^2}\ . \tag{51}$$

Here, to preserve unitarity, the energies in the denominator have been obtained in slight modification of (31) as

$$\omega_\pm'^2 = \frac{1}{2}\left(E_{N\Delta}^2 + (\Gamma/2)^2 + E_\pi^2 \pm \sqrt{(E_{N\Delta}^2 + (\Gamma/2)^2 - E_\pi^2)^2 + 4\boldsymbol{k}^2 C\omega_\Delta}\right)\ . \tag{52}$$

In fig. 7, we compare the results for the three forms of the asymptotic expansion, obtained with eqns. (48) (dash-dotted line), (49) (full line) and (50) (dotted line) to the full calculation from the previous section (dashed line). At zero temperature near the point of maximal mixing $|\boldsymbol{k}| \approx k_\perp$ the first form (48) is rather crude and hence will be avoided henceforth. The approximation (49) however corresponds quite well to the full calculation - while the third form (50) contributes unphysical strength at low energies, at least for the energy independent Γ used here.

While an explicitly energy dependent Γ might cure this (eqn. (46) still holds in this case), for our discussion of an energy-independent Γ we thus consider eqn. (49) the best choice for an approximate treatment of the Δ-width. For zero temperature, this agreement prevails at all momenta up to twice nuclear density, see the upper panel in fig. 9.

However, the picture changes substantially when looking at finite temperatures - and this was the main goal of the present work. We find, that

at finite temperature the asymptotic expansion severely underestimates the smearing of the spectrum. This is shown in the small inserts to fig. 8 and the lower panel of fig. 9: the discrepancy increases with temperature, but is almost independent of density in the range considered here.

The deviation is connected to the smearing of the Fermi surface, i.e. to the quasistatic approximation leading from (41) to (45) rather than to the asymptotic equation (46). It can be corrected by fitting an "effective" $\Gamma'(\boldsymbol{k}, T)$ to the full calculation.

In the main body of fig. 8 and the lower panel of fig. 9 we show the results of such a fit. Both figures contain a comparison of the full calculation from the previous section (thick lines) to the asymptotic expansion (thin lines). The asymptotic expansion has been taken from eqn. (49), but with a modified $\Gamma'(\boldsymbol{k}, T)$ – whereas the small inserts to these figures are calculated with the Γ from table 1.

The effective $\Gamma'(\boldsymbol{k}, T)$ is plotted in fig. 10. Of course, such a momentum and temperature dependence also arises from a medium-dependent Δ spectral function or its momentum dependent parameterization [16]. However, we find that even when starting from eqn. (39) with a constant Γ, the pion propagation feels an effectively medium dependent Γ'. The changes here are of similar magnitude as one would expect from a self-consistent calculation of the Δ-width [11]. The additional width is additive, i.e., if the bare width of the Δ is doubled, the effective width according to our fit does not increase by a factor of two.

It is now clear, how to construct a simple approximation to the pionic spectral function in HCNM. This can be used in the calculation of observable quantities. However, for the practical purpose of simulations as well as for a qualitative understanding, a particle-like picture would be more appealing. We therefore attempt to define an interacting pion propagating in HCNM.

Such description is far from trivial, since one cannot simply look for poles of the pion propagator in the complex plane: they are on an unphysical Riemann sheet. Instead, one has to define the particle-like excitations (which are *not* quasi-particles) on the real energy axis. To this end, we follow a procedure recently outlined for non-relativistic systems [20], and write the inverse retarded propagator for the pions with real energy k_0 as

$$k_0^2 - E_\pi^2(\boldsymbol{k}) - \Pi^R(k_0, \boldsymbol{k}) = (k_0 - (\varepsilon_k - i\gamma_k))\,(k_0 + (\varepsilon_k + i\gamma_k)) \ . \qquad (53)$$

To lowest order, the on-shell energy ε and its imaginary part γ are then given

as the solution of the equation

$$(\varepsilon_k - i\gamma_k)^2 - E_\pi^2(\boldsymbol{k}) - \Pi^R(\varepsilon_k, \boldsymbol{k}) = 0\ . \tag{54}$$

Using the asymptotic expansion (49) after the correction according to (22), the solution of this equation corresponds to solving two coupled (non-linear) polynomial equations. Although this is easily done numerically, one cannot guarantee the existence of a unique solution. Indeed, for a broad range of momenta *two* solutions exist. They are shown as full thin lines in fig. 1, the shaded areas correspond to the regions $\varepsilon_k + \gamma_k > E > \varepsilon_k - \gamma_k$.

The fact, that the real part of the energies ε_k corresponds quite well to the peaks of the full calculation from section 4 proves the validity of the approximation (54). Moreover, as can be seen from fig. 11, the imaginary part of the on-shell energy is comparable to the half-width of the spectral function peaks. Thus we conclude, that the definition (54) is an easy way to determine the properties of pionic modes in HCNM.

It remains to clarify, which of the two branches to chose for a particle-like pion propagation. A natural choice is, to take as "the pion" that solution of eqn. (54), which has the smallest imaginary part of the energy. This choice is unique, since one of the solutions has an imaginary part increasing with momentum, while the other decreases with $|\boldsymbol{k}|$. Thus, there exists a certain momentum, where one has to switch from one branch to the other with increasing or decreasing momentum. As one can find analytically from the above equations, this crossover point is given by the momentum value, where the two residues in the quasistatic zero-width approximation are equal, i.e., by the $k_\perp$ defined in eqn. (34) (cf. also fig. 1 and the vertical lines in fig. 11).

The real and imaginary part of the pion dispersion relation obtained with this description are shown in fig. 11, for several temperatures at 1.69 nuclear density. To keep the picture as clean as possible, we have performed these calculations with the constant Γ from table 1. In fig. 6 we show the same quantities at three different densities.

In short, the peculiarity of our definition is a *jump* in the dispersion relation of the interacting pion (To guide the eye, the curves in the figures have been drawn continuously.). Such a discontinuity in the dispersion relation can be seen as a potential well in momentum space: If the pion momentum gets smaller than the critical value of $k_\perp$, the energy drops by a certain

amount. This drop diminishes with increasing temperature (see fig. 11) and decreasing density (see fig. 6). The energy released in this process is distributed into the medium by the coupling to the broad Δ-hole excitation (cf. fig. 4) .

The imaginary part of the quasi-pion energy has its maximum at the boundary of the well, i.e., at momentum $k_\perp$. As can be derived, this maximum is $max(\gamma_k) = \Gamma/2$. The quasi-pion therefore moves through the nuclear medium with a complex velocity, similar to a photon crossing a semi-transparent medium.

6 Conclusions

In this work, the formalism of Thermo Field Dynamics was applied to the Δ-hole model at finite temperature. The model was analyzed starting from a seemingly simple quasistatic zero Δ-width approximation, which can be treated analytically. However, as an important difference to other treatments of the model, a diminishing of the coupling was found at higher temperature (cf. eqn. (26) and its discussion, and compare to refs. [10, 12]).

Furthermore we could show, that the residue-weighted energy average of the eigenmodes in this simple approximation does not give a proper description of pion propagation: one could as well use the free pion in the system.

We then went to a more elaborate treatment, taking into account a constant width of the Δ_{33} resonance. The numerical calculation of the pionic spectral function at finite temperature was performed to establish a reference point for simplifications. Of these the most promising was found by performing an asymptotic expansion of the Δ spectral function, leading to a polarization function (49) that is a straightforward generalization of the zero-width approximation. This expansion can also be applied to more sophisticated parametrizations of the Δ spectral function, where the width is E, $\boldsymbol{k}$, T - dependent.

Our approximation agrees quite well with the full reference calculation, thus supplying a simple expression for the full pion propagator in hot, compressed nuclear matter (HCNM). Another approximation however was found to produce artificial spectral strength at low energies (cf. eqn. (50), fig. 7 and ref. [19]).

Since the Bose equilibrium distribution function is strongly peaked at

small energies, this strength leads to a large increase of the effective (near-) equilibrium occupation number. While such spectral contribution at small energies might therefore explain the low momentum pion enhancement in relativistic heavy-ion collisions [12, 21], it is clearly not a consequence of a rigorous treatment of the Δ-hole model.

The successful description of phenomena like lepton pair creation in heavy-ion collisions requires to use the full pion propagator according to (12), with one or the other spectral function as discussed in this work [18, 19]. However, for cascade-like simulation calculations as well as for other practical purpose, a description of "the pion" in terms of spectral functions and matrix valued propagators might be impractical.

We therefore gave a simplified description, i.e. defined particle-like excitations with a definite (complex) energy. While the real part of this dispersion relation coincides very well with the simple quasistatic zero-width approximation, our model also gives the spectral width of the pion-like excitation. The real part of the dispersion relation obtained in this way includes a discontinuity, i.e., a potential well in momentum space – and the absorptive part of the dispersion relation is maximal at the boundary of the well. Hence, this simplified description of pionic modes in HCNM corresponds to a kind of "pion-optical" picture.

One of the implications of this picture is, that the (near-)equilibrium occupation number of pions is enhanced for pion momenta just inside the potential well, while it is decreased for momenta just above the step (cf. eqn. (34)). Further implications, e.g. on pion spectra obtained from nuclear collisions, will have to be investigated.

Furthermore one has to add, that a fully self-consistent treatment of the medium- and temperature dependent Δ-width is desirable. Without such considerations, a simple parameterization of our results as function of temperature and density is meaningless. However, since we derived a quite large effect of temperature on the pion width, small changes from such calculation will not modify our conclusions. In essence, one has to solve the Fock problem for pions, nucleons and Δ's - and for this, solutions exist that can be supplemented by the pionic spectral functions we derived [14].

We did not discuss negative energy states and renormalization problems in the present paper. It is nevertheless clear that they play an important role for model consistency: Although the Δ-$\bar{N}$ continuum begins at much higher energies than discussed here, the polarization function has to fulfil

the dispersion relation (16). Hence, even at low energies mesonic spectral functions are influenced by the continuum [22].

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References

[1] Proc. 1st Workshop on Thermal Field Theories and Their Applications eds. K.L.Kowalski, N.Landsman, Ch.G. van Weert, Physica **158 A** (1989) 1

[2] *Thermal Field Theories*, Proc. 2nd Workshop on Thermal Field Theories and Their Applications eds. H.Ezawa, T.Arimitsu and Y.Hashimoto (North Holland, Amsterdam 1991)

[3] N.P.Landsman, Ann.Phys. **186** (1988) 141

[4] J.Schwinger, J.Math.Phys. **2** (1961) 407 L.V.Keldysh, JETP **20** (1965) 1018

[5] S.Mrowczynski and U.Heinz, U Regensburg preprint (1992)

[6] T.Arimitsu and H.Umezawa, Prog.Theor.Phys. **77** (1987) 32 and 53

[7] H.Umezawa, *Advanced Field Theory: Micro, Macro and Thermal Physics*, (American Institute of Physics, in press 1993)

[8] P.A.Henning and H.Umezawa, *Diagonalization of Propagators in TFD for Relativistic Quantum Fields* GSI-Preprint 92-61 (1992), subm. to Nucl.Phys. **B**
Diagonalization of full finite temperature Green's functions by quasi-particles Phys.Lett. **B** (1993) in press

[9] G.E.Brown and W.Weise, Phys.Rep. **22** (1975) 279

[10] W.Ehehalt, Gy.Wolf, W.Cassing et. al., Phys.Lett. **B298** (1993) 31

[11] C.M.Ko, L.H.Xia and P.J.Siemens, Phys.Lett. **B 231** (1989) 16

[12] L.Xiong, C.M.Ko and V.Koch, Phys.Rev. **C47** (1993) 788

[13] H.Umezawa, H.Matsumoto and M.Tachiki,
Thermo Field Dynamics and Condensed States
(North-Holland, Amsterdam 1982)

[14] P.A.Henning, Nucl.Phys. **A546** (1992) 653

[15] A.Rittenberg et.al., Rev.Mod.Phys. **43** (1971) 5114
Y.Kitazoe, M.Sano, H.Toki and S. Nagamiya, Phys.Lett. **B 166** (1986) 35

[16] J.H.Koch, E.J.Moniz and N.Ohtsuka, Ann.Phys. **154** (1984) 99

[17] M.Schmidt, G.Röpke and H.Schulz, Ann.Phys.**202** (1990) 57

[18] M.Herrmann, B.Friman and W.Nörenberg
Properties of the ρ-meson in dense hadronic matter
GSI-Report 92-10 (1992) and GSI-Preprint 93-12 (1993), subm. to Nucl.Phys. **A**

[19] M.Asakawa, C.M.Ko, P.Levai and X.J.Qiu, Phys.Rev. **C46** (1992) 1159
C.M.Ko, P.Levai and X.J.Qiu,
The ρ-meson in dense hadronic matter, preprint (1993)

[20] H.Chu and H.Umezawa
Stable Quasi-Particle Picture in Thermal Quantum Field Physics
University of Alberta Preprint (1993)

[21] G.Odyniec et al., in Proc. of the 8th High Energy Heavy-Ion Study, edited by J.Harris and G.Woznick, LBL Report No. 24580, p. 215

[22] J. Diaz Alonso, B.Friman and P.A.Henning,
Normal Modes in Nuclear Matter at Finite Temperature
in preparation.

Figure 1: Pionic dispersion relation at $\rho_b = 1.69$ nuclear density.
Full thick lines: Quasistatic zero-width approximation, eqn. (29)
Dotted lines: Free energies E_π and $E_{N\Delta}$, vertical lines $k_\perp$ from eqn. (34)
Dash-dotted lines: $Z_\pm$ weighted energies, eqn. (36)
Dots: Location of the peaks in the spectral function of the full calculation, (42).
Full thin lines and hatched area: Asymptotic expansion with on-shell definition, eqn. (54). Hatched is the region within $\varepsilon_k \pm \gamma_k$.

Figure 2: Pionic spectral function at $|\boldsymbol{k}| = 50$ MeV, $\rho_b = 1.69$ nuclear density.

Figure 3: Pionic spectral function at $|\boldsymbol{k}| = 200$ MeV.

Figure 4: Pionic spectral function at $|\boldsymbol{k}| = 350$ MeV.

Figure 5: Pionic spectral function at $|\boldsymbol{k}| = 500$ MeV.

Figure 6: Pionic spectral function at $|\boldsymbol{k}| = 650$ MeV.

Figure 7: Pionic spectral function at T=0 and $\rho_b = 1.69$ nuclear density
Full line: Asymptotic expansion, eqn. (49)
Dashed line: Full calculation from section 4
Dotted line: eqn. (50), Dash-dotted line: eqn. (48).

Figure 8: Pionic spectral function at T=0.1 GeV and ρ_b= 1.69 nuclear density
Thick lines: Full calculation from section 4
Thin lines: Asymptotic expansion, eqn. (49)
Temperatures 50 MeV (full lines), 100 MeV (dashed) and 150 MeV (dash-dotted)
Main panel calculated with $\Gamma'(\boldsymbol{k}, T)$ from fig. 10.
Insert calculated with Γ=0.12 GeV.

Figure 9: Pionic spectral function at T=0 and T=0.1 GeV

Thick lines: Full calculation from section 4

Thin lines: Asymptotic expansion, eqn. (49)

ρ_b= 1.69 (full lines), 0.92 (dashed) and 0.47 (dash-dotted) nuclear density.

Lower main panel calculated with $\Gamma'(\boldsymbol{k}, T)$ from fig. 10.

Top panel and insert calculated with Γ=0.12 GeV.

Figure 10: Effective thermal Δ-width $\Gamma'(\boldsymbol{k}, T)$ in GeV at $\rho_b = 1.69$ nuclear density.
Both panels show the same data, contour spacing in the lower panel is 10 MeV.

Figure 11: Pion dispersion relation at $\rho_b = 1.69$ nuclear density.
Upper panel real part ε_k, lower panel imaginary part γ_k of the effective pion energy from (54).
Temperatures 1 MeV (full lines), 50 MeV (dashed), 100 MeV (dash-dotted), 150 MeV (dash-double-dotted) and 200 MeV (dotted).
Full thin line: free pion, vertical lines $k_{\pm}$ from eqn. (34).

Figure 12: Pion dispersion relation at T=0 and T=0.1 GeV.

Upper panel real part ε_k, lower panel imaginary part γ_k of the effective pion energy from (54).

Temperatures 1 MeV (thin lines) and 100 MeV (thick lines).

Baryon densities ρ_b= 1.69 (full lines), 0.92 (dashed) and 0.47 (dash-dotted) nuclear density.

Dotted line: free pion